GRADIENT DAMAGE MODELS COUPLED WITH PLASTICITY

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It is now well established that gradient damage models are very efficient to account for the behavior of brittle and quasi-brittle materials. Their basic ingredients are: (i) a decreasing dependency of the stiffness $\mathsf{E}(\alpha)$ on the damage variable α; (ii) no more rigidity at the ultimate damage state (say $\mathsf{E}(1) = 0$); (iii) a critical stress σ_c; (iv) a softening behavior with a decrease of the stress from σ_c to 0 when the damage goes to 1; (v) a gradient damage term in the energy which necessarily contains an internal length ℓ and which limits the damage localization. Accordingly, the process of crack nucleation is as follows [3]: (i) a first damage occurs when the stress field reaches the critical stress somewhere in the body; (ii) then, because of the softening character of the material behavior, damage localizes inside a strip the width of which is controlled by the internal length ℓ; (iii) the damage grows inside this strip, but not uniformly in space (the damage is maximal at the center of the strip and is continuously decreasing to 0 so that to match with the undamaged part of the body at the boundary of the strip); (iv) a crack appears at the center of the strip when the damage reaches there its ultimate value (say $\alpha = 1$). During this crack nucleation process, some energy is dissipated inside the damage strip and this dissipated energy involves a quantity $\mathsf{G_c}$ which can be considered as the effective surface energy of Griffith's theory. Therefore, $\mathsf{G_c}$ becomes a byproduct of the gradient damage model which can be expressed in terms of the parameters of the model (specifically, $\mathsf{G_c}$ is proportional to $\sigma_c^2\ell/\mathsf{E}(0)$).

However, this type of "quasi-brittle" models are not able to account for residual strains and consequently cannot be used in ductile fracture. Moreover there is no discontinuity of the displacement in the damage strip before the loss of rigidity at its center, *i.e.* before the nucleation of a crack. In other words such a model cannot account for the nucleation of cohesive cracks, *i.e.* the existence of surface of discontinuity of the displacement with a non vanishing stress. The natural way to include such effects is to introduce plastic strains into the model and to couple their evolution with damage evolution. Of course, this idea is not new and a great number of damage models coupled with plasticity have been developed from the eighties in the spirit of [2]. But our purpose is to construct such models in a softening framework with gradient of damage terms and to see how these models can account for the nucleation of cracks in presence of plasticity. In our knowledge, the previous works are not able to go so far. Here we will adopt a variational approach in the spirit of our previous works [1]. The main ingredients are the following ones: (i) one defines the total energy of the body in terms of the state fields which include the displacement field and the internal variable fields, namely the damage, the plastic strain and the cumulated plastic strain fields; (ii) one postulates that the evolution of the internal

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variables is governed by the three principles of *irreversibility*, *stability* and *energy balance*. In particular, the stability condition is essential as well for constructing the model in a rational and systematic way as for obtaining and proving general properties. Besides, we have the chance that the variational approach works and has been already developed both in plasticity and in damage mechanics, even though only separately up to now. So, it "suffices" to introduce the coupling by choosing the form of the total energy to obtain, by virtue of our plug and play device, a model of gradient damage coupled with plasticity. A part of our paper will be devoted to this task. Specifically, our model, presented here in a three-dimensional setting, contains three state functions, namely $\mathsf{E}(\alpha)$, $\mathsf{d}(\alpha)$ and $\bar{\sigma}_{\mathsf{P}}(\alpha)$ which give the dependence of the stiffness, the local damage dissipated energy and the plastic yield stress on the damage variable. So, our choice of coupling is minimalist in the sense that it simply consists in introducing this dependence of the yield plastic stress $\sigma_{\mathsf{P}}(\alpha)$ on the damage variable (with the natural assumption that $\sigma_{\mathsf{P}}(\alpha)$ goes to 0 when the damages goes to 1). In turn, by virtue of the variational character of the model, the product $\sigma_{\mathsf{P}}'(\alpha)\bar{p}$ of the derivative of the state function $\sigma_{\mathsf{P}}(\alpha)$ by the cumulated plastic strain $\bar{p}$ enters in the damage criterion and this coupling plays a fundamental role in the nucleation of a cohesive crack.

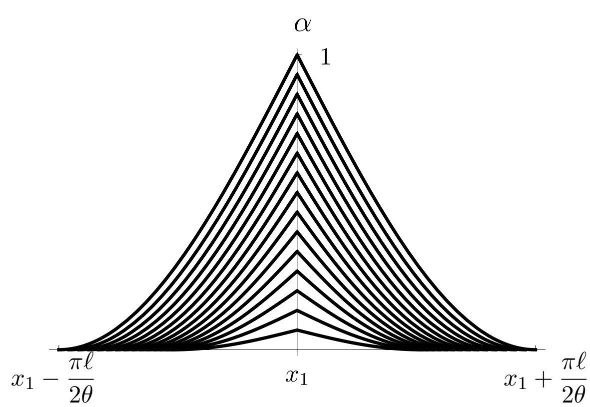

FIGURE 1. Localization damage process with nucleation of a cohesive crack at the center of the damage zone

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